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SUMMARY

A method is described of rapidly estimating the position of the laminar separation point from the given pressure distribution along a body; the method is applicable to a fairly wide variety of cases. The laminar separation point is found by the von Kármán-Millikan method for a series of velocity distributions along a flat plate, which consist of a region of uniform velocity followed by a region of uniformly decreasing velocity. It is shown that such a velocity distribution can frequently replace the actual velocity distribution along a body insofar as the effects on laminar separation are concerned.

An example of the application of the method is given by using it to calculate the position of the laminar separation point on the N.A.C.A. 0012 airfoil section at zero lift. The agreement between the position of the separation point calculated according to the present method and that found from more elaborate computations is very good.

INTRODUCTION

Of the various available methods of calculating the characteristics of the laminar boundary layer in two-dimensional flow and, in particular, the position of the laminar separation point, the von Kármán-Millikan method (reference 1) seems to be the most reliable. This method, when applied to the boundary-layer flow about an elliptic cylinder, showed good agreement with experiment when other methods failed (reference 2). The von Kármán-Millikan method, however, has the disadvantage that the computations are usually cumbersome and lengthy. Because of the growing importance of the boundary-layer problem, some method of rapidly finding the position of the laminar separation point is needed.

Although the calculations given in reference 1 for a family of "double-roof" velocity distributions, consisting of a region in which the outside velocity increases linearly with distance along the surface followed by a region in which the velocity decreases linearly, partly satisfy the need, some easy method having more general applicability is required.

The purpose of the present paper is to develop a more general method of rapidly estimating the position of the laminar separation point. The effects of an adverse velocity gradient on laminar separation are studied.

The method of estimating the position of the laminar separation point presented in this report is applicable to bodies having velocity distributions whose effects can be approximated by a region of uniform velocity followed by a region of uniformly decreasing velocity. It is to be noted that the boundary-layer velocity distribution at the point along the surface of a body where the outside velocity is a maximum is very nearly the same as the Blasius distribution for a flat plate. Thus, the condition of the boundary layer at the point of maximum velocity can be reproduced by the flow over an equivalent length of a flat plate with uniform velocity equal to the maximum velocity. In a large number of cases, the velocity distribution back of the point of maximum velocity can be well represented by a straight line up to the laminar separation point. The present method therefore has fairly general applicability.

The laminar separation point is first calculated by the von Kármán-Millikan method for a series of velocity distributions over a flat plate; each distribution consists of a region of uniform velocity followed by a region of uniformly decreasing velocity. The velocity decrement sufficient to cause separation is then found as a function of the velocity gradient, which is expressed nondimensionally. The decrement varies from 10.2 percent of the maximum velocity to zero, depending on the value of the gradient.

The relation of conditions on an airfoil section to those for which the calculations were carried out is discussed. As an example of the application of the method, the position of the laminar separation point on the N.A.C.A. 0012 airfoil section at zero lift is calculated.

CALCULATIONS AND RESULTS

Throughout the present calculations, the reference length s_o is taken as the distance from the leading edge of the plate to the point at which the adverse velocity gradient is applied. The reference velocity U_o is taken as the velocity outside the boundary layer in the uniform-velocity region over the plate.

The velocity gradient F can then be expressed in the following nondimensional form:

$$F = \frac{s_o}{U_o}\frac{dU}{ds}$$

where U is the velocity outside the boundary layer at any point along the surface.

s, the distance along the surface.

The following relations give the velocity distribution outside the boundary layer over the plate:

$$\frac{U}{U_o} = 1 \qquad \text{from} \quad \frac{s}{s_o} = 0 \quad \text{to} \quad \frac{s}{s_o} = 1$$

and

$$\frac{U}{U_o} = 1 + F\left(\frac{s - s_o}{s_o}\right) \qquad \text{for} \quad \frac{s}{s_o} \geqq 1$$

Figure 1 shows the form of the velocity distribution over the plate for several values of F.

The position of the laminar separation point was calculated by the method of reference 1 for a series of values of F. In each case the velocity decrement $\Delta U/U_o$ sufficient to cause separation was found. The results of these calculations are given in figure 2 as a plot of $\Delta U/U_o$ or U_s/U_o against F, where U_s is the velocity outside the boundary layer at the separation point.

DISCUSSION

Figure 2 shows that small absolute values of the velocity gradient F correspond to large values of the velocity decrement $\Delta U/U_0$ and that large values of F correspond to small values of $\Delta U/U_0$. When F is equal to zero, the outside velocity can be reduced by 10.2 percent of the maximum velocity U_0 before separation occurs. As the value of F increases, the amount by which the velocity can be reduced after reaching its maximum value approaches zero.

It is important to note that the magnitude of the dimensional quantity dU/ds is not in itself sufficient to determine how much the velocity can be reduced from the maximum value before separation occurs. Figure 3 shows the effect of applying a given velocity gradient dU/ds at several distances from the leading edge. The amount of the reduction from the maximum value before laminar separation occurs is much greater when the gradient is applied close to the leading edge. If the gradient is applied immediately at the leading edge, the velocity decrement is independent of dU/ds. For this case F is always zero.

When an attempt is made to apply the results of the foregoing calculations to an airfoil, the effects produced by the actual velocity distribution over the body must be analyzed and compared with those produced by the assumed type of velocity distribution.

It is shown in reference 3 that, at any point along the surface of a body at which dp/ds is equal to zero (where p is the pressure), the curvature of the laminar boundary-layer profile at the surface is zero. This relation is applicable at the point of maximum velocity along a body as well as to a flat plate with uniform velocity. Such considerations and comparisons with calculations of actual boundary-layer profiles by the von Kármán-Millikan method indicate that the shape of the boundary-layer profile at the point of maximum velocity outside the boundary layer is nearly the same as the Blasius flat-plate profile. Because any boundary-layer profile is specified completely by its shape and thickness, the effect of any region of rising velocity is only to affect the boundary-layer thickness at the point of maximum velocity. This effect can be reproduced by a flow of uniform velocity over a suitable length of flat plate.

The velocity distribution along the surface back of the point of maximum velocity can usually be approximated with sufficient accuracy by a straight line. The approximation need be valid only as far as the point at which laminar separation occurs.

Because a favorable pressure gradient makes for thinner boundary layers, the equivalent length of flat plate is somewhat less than the actual distance from the forward stagnation point of a body to the position of maximum velocity. The velocity over the plate is assumed equal to the peak velocity over the body. If boundary-layer measurements are available at any point P near the point of maximum velocity, the length of plate equivalent to the region upstream of P is $s_1 = \delta R_\delta/(5.53)^2$

where δ is the boundary-layer thickness.

$R_\delta = U\delta/\nu$, Reynolds Number based on the boundary-layer thickness.

ν, the kinematic viscosity.

The distance s_0 is then equal to s_1 plus the distance from P to the point s_2 at which the adverse velocity gradient is applied. (See fig. 4.)

When no suitable boundary-layer measurements are available, the equivalent length of flat plate can be found from the following approximate relation:

$$\frac{s_1}{c} = \int_0^{s/c} \left(\frac{U}{U_0}\right)^{8.17} d\left(\frac{s}{c}\right) \qquad (1)$$

where U_0 is the peak velocity along the surface.

c, the airfoil chord.

The integration is to be carried out over the region extending from the forward stagnation point to the position of maximum velocity. In this case also, $s_0 = s_1 + s_2$.

The foregoing relation for the equivalent length of flat plate was derived from the assumption that the velocity distribution in the boundary layer over the portion of the surface upstream of the point of maximum velocity is similar to the Blasius flat-plate distribution and from the substitution of this assumption in the von Kármán momentum equation.

The integration indicated in equation (1) can, in general, be graphically performed. This graphical integration has been carried out for a series of velocity distributions similar to the distribution over the forward portion of the N.A.C.A. 0012 airfoil section at zero lift. For curves of this series, it was found that

$$\frac{s_1}{s_m} = 0.376 \left(\frac{\Delta p}{q_m}\right)^{-0.164} \tag{2}$$

where s_m is the actual distance along the surface from the forward stagnation point to the point of minimum pressure.

p, the difference in pressure between the points $s_m/2$ and s_m.

q_m, the local dynamic pressure at s_m.

Equation (2) is valid for $0.003 < \dfrac{\Delta p}{q_m} < 0.2$.

From the two straight lines representing the assumed velocity distribution, which replaces the actual velocity distribution over the body, the value of F is computed:

$$F = \frac{s_o}{U_o}\frac{dU}{ds}$$

or

$$F = \frac{-s_o}{2(1 - P_m)}\frac{dP}{ds}, \text{ approximately}$$

where $P = (p - p_\infty)/q$ is the pressure coefficient.

p_∞, the static pressure in the undisturbed stream.

p, the static pressure at any point on the airfoil.

q, the dynamic pressure based on the free-stream velocity.

P_m, the pressure coefficient at the point of minimum pressure.

$\frac{dP}{ds}$, the slope of the line approximating the pressure distribution back of the point of minimum pressure.

The corresponding value of U_s/U_o is found from figure 2. Separation is calculated to occur at the point where U falls to the value $U_o(U_s/U_o)$.

As an example, the method has been used to calculate the position of the laminar separation point on the N.A.C.A. 0012 airfoil section at zero lift. The velocity distribution over the airfoil in terms of the free-stream velocity V was found by the method of reference 4. Figure 4 shows that, for unit chord, the position of the point of maximum velocity is $s_m = 0.125$ and that $\Delta p/q_m = 0.037$. Hence, from equation (2),

$$\frac{s_1}{s_m} = 0.376\ (0.037)^{-0.164} = 0.646$$

and $s_1 = 0.0807$. The position along the surface at which the adverse gradient is applied is $s = 0.140$. The distance s_2 is then equal to $0.140 - 0.125 = 0.015$. The total equivalent length of flat plate is $s_o = s_1 + s_2 = 0.0957$. The nondimensional velocity gradient is

$$F = \frac{s_o}{U_o}\frac{dU}{ds} = -\frac{0.0957}{1.190}\ (0.243) = -\ 0.0196$$

From figure 2, $U_s/U_o = 0.919$, and the velocity at the separation point is $1.190 \times 0.919 = 1.094$. Separation is calculated to occur at $s/c = 0.536$, the position at which U/V falls to the value 1.094.

The position of the laminar separation point was also computed according to the von Kármán-Millikan method. Two power series were used to approximate the velocity distribution. These calculations indicated that the separation point was situated at $s/c = 0.55$. The agreement between the two sets of calculations is considered especially satisfactory in view of the simplicity of the present method.

The method was also applied to the case of the elliptic cylinder reported in reference 2. The previous calculations showed that separation was to be expected at a distance s along the surface of 1.92. The present method gave $s = 2.00$ as the separation point. The experimentally observed position was $s = 1.99$.

CONCLUDING REMARKS

A fairly general method of rapidly estimating the position of the laminar separation point, based on the von Kármán-Millikan boundary-layer theory, has been devised. Good agreement was obtained between the results of this method and those of more elaborate computations when it was applied to calculate the laminar separation point on the N.A.C.A. 0012 airfoil section at zero lift.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., September 9, 1938.

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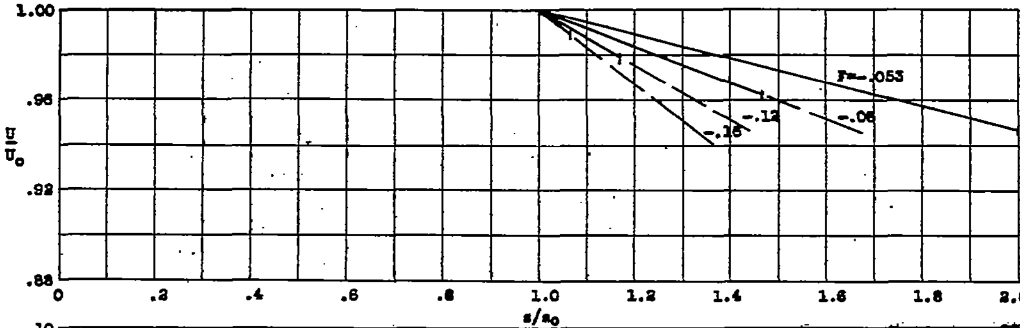

Figure 1.- Typical velocity distributions to which the calculations are strictly applicable. The ticks on the curves show the calculated positions of the laminar separation points.

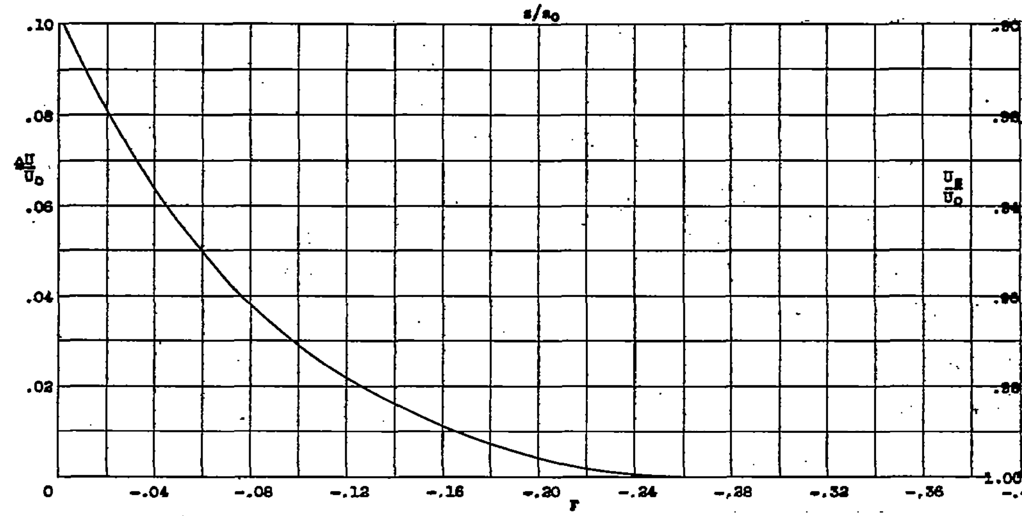

Figure 2.- Curve showing the relation between the velocity decrement $\Delta U/U_0$ obtainable before laminar separation occurs and the velocity gradient F.

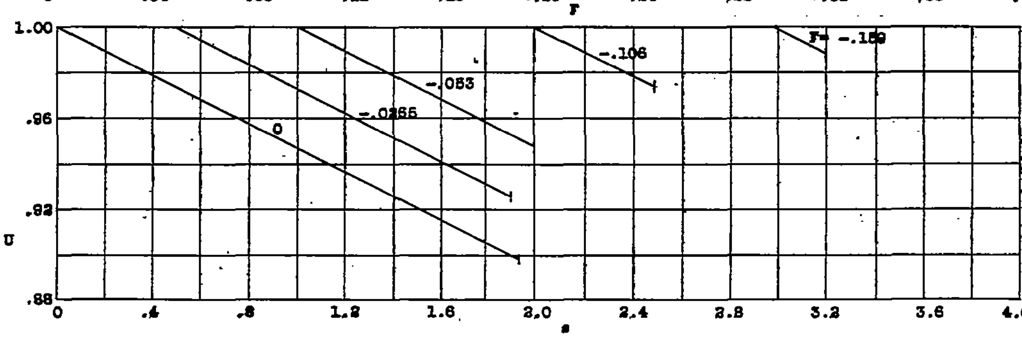

Figure 3.- Effect on laminar separation of applying the same velocity gradient at various distances from the leading edge.

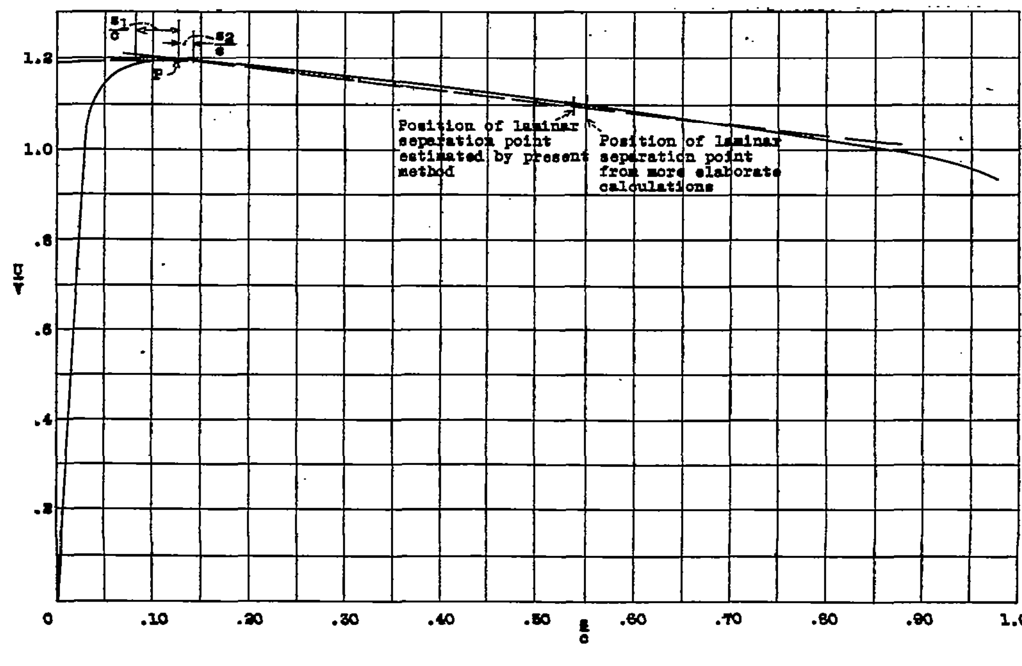

Figure 4.- Comparison of present method of estimating the position of the laminar separation point on the N.A.C.A. 0012 airfoil with a method using more elaborate computations.